FALL 2003 MATH CONTEST

1. Let the operation * be defined a *b *c = $a^b - b^c + c^a$. What is the value of 1 * (-1) * 2?

2. Two days ago Rebecca was 13 years old; next year she will be 16. When is her birthday?

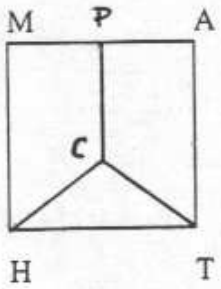
3. A Euclidean buys a stereo for 2330 Euclidean dollars, pays with a bill worth 3000 Euclidean dollars, and receives 340 Euclidean dollars in change. What positive number base do the Euclideans use?

4. Three people play a game with the understanding that the loser is to double the money of each of the other two. After three games each has lost just once and each ends up with $24. With how much money did each one start?

5. I bought a bronco down in Texas for $24. After paying his keep for awhile, I sold him for $60. That looked profitable, but after the cost of his keep, I had lost an amount equal to half the cost of what I paid for him plus one-fourth of the cost of his keep. How much did I lose?

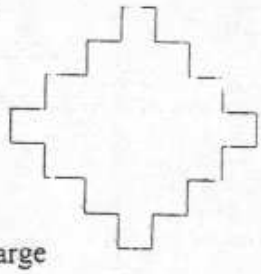
6. Misters Blue, White, and Gray have shirts and ties that are blue, white and gray. No man's clothing is the same as his name. If Mr. Blue's tie is the same color as Mr. Gray's shirt, what is the color of Mr. White's shirt? **(Each color is used exactly once for shirts and ties.)**

7. In square MATH, MA = 16, CP ⊥ MA, and CH = CT = CP = x. Then x = ?



8. In the polygon shown each side is perpendicular to its adjacent sides, and all 28 of the sides are congruent. The perimeter of the polygon is 56. The area of the region bounded by the polygon is:

A. 84 B. 96 C. 100 D. 112



9. The two digit numbers from 19 to 92 are written consecutively to form the large integer

N = 192021222324...909192.

If 3^k is the highest power of three that is a factor of N, then k = ___?

A. 0 B. 1 C. 2 D. more than 2

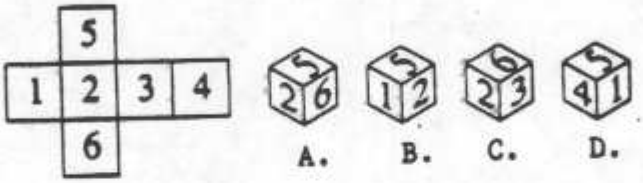
10. A sample consisting of five observations has an arithmetic mean of 10 and a median of 12. The smallest value that the range (largest observation minus the smallest observation) can assume is

A. 2 B. 5 C. 7 D. 14

11. An 8 in. by $2\sqrt{2}$ in. rectangle has the same center as a circle of radius 2 in. The area of the region common to both is:

A. 2π sq. in. B. $(2\pi + 2)$ sq. in. C. $(4\pi - 4)$ sq. in. D. $(2\pi + 4)$ sq. in.

12. The figure on the left is folded along the edges of the squares to make a solid as shown. Which is the correct solid for the given array?



13. The King Theater and the Star Theater run movies continuously, and each starts its first feature at 1:00 PM. If the movie shown at the King lasts 80 minutes and the movie at the Star lasts two hours, when will the two movies start at the same time again?

14. A group of twelve strangers sat in a circle and each one got acquainted only with the person to the left and to the right. Then all twelve people stood up and each one shook hands (once) with each of the others who was still a stranger. How many handshakes occurred after all stood up?

A. 24 B. 36 C. 45 D. more than 45

15. A frog falls into a well that is 18 feet deep. Each day the frog jumps up a total distance of 6 feet. At night, however, as the frog grips the slimy well walls, he slips down by four feet. At this rate, how many days will it take frog to jump to the rim of the well?

16. Find the exact value of the following expression.

$$\sqrt[3]{2}\cdot\sqrt[12]{4}\cdot\sqrt[36]{8}\cdot\sqrt[96]{16}\cdot\sqrt[240]{32}\cdot\sqrt[576]{64}\ldots$$

17. The graph of which of the following equations has the highest point?

A. $y = \pi - x^2$ B. $x^2 + y^2 = 9$ C. $y = \sqrt{(10 - x^2)}$ D. $y = -8(x + 7)^2 + 19/6$

18. If $24^{x+y}\,9^{x-y} = 4^{10}6^{40}$, then the value of $x^2 - y^2$ is :

A. less than 100 B. 100 C. 200 D. None of the others

19. Find the exact value of $x = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}$

20. A data set consists of n pieces of data, not necessarily all different. Its mean is M. Suppose that the largest piece of data, 8M/3, and the two smallest pieces of data, M/6 and M/2 are deleted from the data set. Find in terms of M and n, the mean of the data set obtained by dropping these three pieces of data.

NYSMATYC MATH LEAGUE COMPETITION
Fall 2003

ANSWER SHEET

Directions: You have one full hour to take this test. Scrap paper is allowed. The use of a calculator is permitted, but not stored programs on the calculator. Moreover, books, tables, and computers are not permitted. You are not expected to answer all problems. Yet, don't waste too much time on any one problem. Four points are awarded for each correct answer, one point is deducted for each incorrect answer and nothing is deducted for a blank. No partial credit. Note: Answers may be given in fractional form, radical form, or as an approximation to the third decimal place as appropriate. (For example, $\sqrt{7}$ or 2.646)

Name KEY **College** _____

Home Address _____
(include city, state, zip code)

Name of teacher in whose class you are enrolled _____

1.	2	11.	D $(2\pi+4)$ IN.^{2}
2.	12|31	12.	B
3.	7	13.	5 PM
4.	\$12, \$21, \$39	14.	D
5.	\$64	15.	7 DAYS
6.	BLUE	16.	$2^{2/3}$
7.	10 UN.	17.	D
8.	100 UN.^{2} C	18.	C 200
9.	B 1	19.	$\frac{1+\sqrt{5}}{2}$
10.	B 5	20.	$\frac{M(3N-10)}{3(N-3)}$

Correct _____ X 4 = _____

Incorrect _____ X –1 = _____

Total _____

SOLUTIONS
FALL 2003

1.) $1 * (-1) * 2 = 1^{-1} - (-1)^2 + 2^1$
$= 1 - 1 + 2 =$ **2 ANS.**

2) REBECCA WAS BORN ON 12/31/89
THE DAY IS 1/1/04. 2 DAYS EARLIER
SHE WAS 13, THEN 14 ON 12/31/03. THE
NEXT YEAR IS 2005 IN WHICH
SHE WILL BE 16. ANS. 12/31

3) LET a BE THE BASE, THEN
$3000 = 3a^3$; $2330 = 2a^3 + 3a^2 + 3a$
AND $340 = 3a^2 + 4a$
$3000 - 2330 = 340$ SO
$3a^3 - (2a^3 + 3a^2 + 3a) = 3a^2 + 4a$
$3a^3 - 2a^3 - 3a^2 - 3a = 3a^2 + 4a$
$a^3 - 3a^2 - 3a = 3a^2 + 4a$
$a^3 - 6a^2 - 7a = 0$
$a(a^2 - 6a - 7) = 0$
$a(a-7)(a+1) = 0$
ONLY POSITIVE ROOT IS **a = 7**

4) CALL THE PLAYERS A, B + C. BY WORKING BACKWARDS, SAY A LOST 3rd GAME THEN HE HAD 1/2 THE MONEY OF THE OTHER 2.
SO ADD \$24 TO A AND SUBTRACT \$12 FROM OTHER TWO.

		A	B	C
AFTER GAME 3		24	24	24
" " 2		48	12	12
LET B LOSE 2 SO	1	24	42	6
LET C LOSE 1 SO	START	12	21	39.

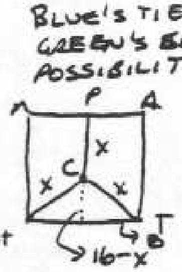
CHANGING THE ORDER JUST MOVES THE AMOUNTS AROUND. SO THE START IS \$12, \$21, AND \$39

5) LET K = COST OF KEEP.
THEN THE LOSS IS
$K - (60 - 24) = \frac{1}{2}(24) + \frac{1}{4}K$
$K - 36 = 12 + \frac{1}{4}K$
$\frac{3}{4}K = 48$
$K = \$64$

6) THE POSSIBLE CASES ARE BELOW

NAME	BL	GR	WH
TIE	G (W)	B W	B G
SHIRT	W G	(W) B	G (B) ANS BLUE

BLUE'S TIE IS THE SAME COLOR AS GREEN'S SHIRT, SO THE ONLY POSSIBILITY IS BOTH WHITE.

7) 

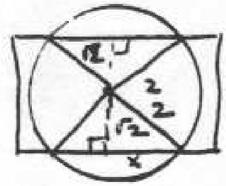
COMPLETING CP TO REACH SIDE HT, WE HAVE A RT. Δ. SO
$x^2 = (16-x)^2 + 8^2$
$x^2 = 256 - 32x + x^2 + 64$
$0 = 320 - 32x$

8) THERE ARE 28 SIDES WHOSE PERIMETER IS 56.
EACH SIDE IS 2 UN. WE CAN THINK OF THE FIGURE AS ROWS OF 2x2 SQUARES.

THE MIDDLE HAS 7, THERE ARE 2 FIVE BLOCK ROWS, 2 @ 3 BLOCKS AND 2 @ 1 BLOCK. $7 + 2(5) + 2(3) + 2(1) = 25$
AREA OF 1 BLOCK = 4 (ANS 4(25) = 100 UN)

9) TO DETERMINE THE FACTORS OF 3 WE NEED TO KNOW THE SUM OF THE DIGITS. THE DIGITS 0, 1, 2, 3, ..., 9 ADD TO $\frac{9 \cdot 10}{2} = 45$.
SO ADDING
TEENS + 20's + 30's + 40's + 50's + 60's + 70's + 80's + 9...
$10 + 10(2) + 45 + 10(3) + 45 + 10(4) + 45 + 10(5) + 45 +$
$10(6) + 45 + 10(7) + 45 + 10(8) + 45 + \frac{3}{8}...$
$= 40 + 7(45) + 10(2+3+4+5+6+7+8)$
$= 40 + 315 + 10(35) = 355 + 350 = 705$
SINCE 705 IS DIVISIBLE BY 3 AND NOT 9 THERE IS EXACTLY ONE FACTOR OF 3.
ANS. 1 **B**

10) SINCE THE MEAN IS 10, THE FIVE NUMBERS MUST ADD TO 5(10) = 50. THE MEDIAN, 12, IS ONE OF THE NOS. SO THE OTHERS ADD TO 38.
IF WE LET 3 OF THE NUMBERS BE 12, THEN THE OTHER 2 ADD TO 50 − 3(12) = 14.
TO KEEP RANGE SMALL, MAKE BOTH 7
SO RANGE = 12 − 7 = 5 **ANS. B**

11) 

DROPPING A ⊥ TO BASE OF RECTANGLE GIVES $x^2 + (\sqrt{2})^2 = 2^2$
$x^2 + 2 = 4$
$x^2 = 2$ $x = \sqrt{2}$

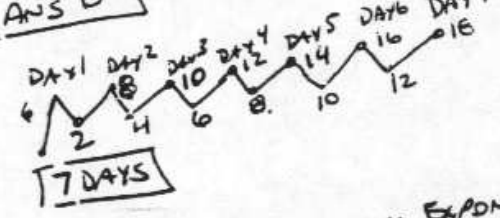
SO AREA OF TWO Δ'S IS
$2(\frac{1}{2})(\sqrt{2})(2\sqrt{2}) = 4$, THE ISOSCELES Δ'S HAVE BASE ANGLES 45°, SO THE REMAINING ANGLE AT THE CENTER IS 90°.
SO THE AREA OF THE 2 SECTORS IS HALF THE AREA OF THE CIRCLE.
SO $A = \frac{1}{2}\pi(2)^2 = 2\pi$
· ANS. $(4 + 2\pi)$ UN.^{2} **D**

12) WHEN IT IS FOLDED THE 5 AND 6 ARE TOP & BOTTOM
THE BOTTOM OF THE FIVE IS ABOVE THE TOP OF 2.
THERE IS ONLY 1 CHOICE, **B.**

13) WE NEED TO FIND THE LEAST COMMON MULTIPLE OF 80 + 120 WHICH IS 240.
SO IT WILL BE 240 MIN = 4 HRS.
ANS. **5PM**

) SINCE EACH PERSON MEETS THE PERSON ON LEFT AND RIGHT, HE KNOWS 3 PEOPLE. THERE ARE 9 TO MEET SO FOR A HANDSHAKE THERE ARE 12 CHOICES FOR THE 1ST AND 9 FOR THE 2ND. BUT IF A SHAKES HANDS WITH B, THEN B HAS MET A. SO $\frac{12\cdot 9}{2} = \frac{108}{2} = 54$

ANS D

15) 

7 DAYS

16) REWRITING WITH FRACTIONAL EXPONENTS

$2^{1/3}\left(2^{2}\right)^{1/12}\left(2^{3}\right)^{1/36}\left(2^{4}\right)^{1/96}\left(2^{5}\right)^{1/200}\left(2^{6}\right)^{1/576}\cdots$

$= 2^{1/3}\cdot 2^{1/6}\cdot 2^{1/12}\cdot 2^{1/24}\cdot 2^{1/48}\cdot 2^{1/96}\cdots$

WE NEED TO ADD $S = \frac{1}{3}+\frac{1}{6}+\frac{1}{12}+\frac{1}{24}+\frac{1}{48}+\frac{1}{96}+\cdots$

$S = \frac{1}{3}\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\right)$

SO $S = \frac{\frac{1}{3}}{1-\frac{1}{2}} = \frac{2}{3}$ ANS.

ANS $2^{2/3}$

17) A HAS HEIGHT π B. CIRCLE RADIUS 3 HEIGHT 3

C. SEMICIRCLE RADIUS $\sqrt{10}$ D. PARABOLA WITH HEIGHT $\frac{18}{6} = 3.1666$

$\sqrt{10} \doteq 3.16227$ $\pi \doteq 3.14159$

ANS. D

18) $4^{10}\cdot 6^{40} = (2^2)^{10}\cdot(2\cdot 3)^{40}$

$= 2^{20}\cdot 2^{40}\cdot 3^{40}$

$= 2^{60}\cdot 3^{40}$ BUT $2^3 = 8$

$= (2^3)^{20}\cdot 3^{20}\cdot 3^{20}$

$= (8\cdot 3)^{20}\cdot 9^{10}$

$24^{x+y}\cdot 9^{x-y} = 24^{20}\cdot 9^{10}$

$x+y = 20$ $x-y = 10 \Rightarrow x^2-y^2 = 200$

19) LET $x = 1+\cfrac{1}{1+\cfrac{1}{1+\cdots}}$

SO $x = 1+\frac{1}{x}$

OR $x^2 = x+1$

$x^2 - x - 1 = 0$

$x = \frac{1\pm\sqrt{1+4}}{2} = \frac{1\pm\sqrt{5}}{2}$

x IS CLEARLY POSITIVE. SO

$x = \frac{1+\sqrt{5}}{2}$

20) THE SUM OF THE n PIECES IS Mn

DELETING 3 SCORES WE HAVE NEW SUM:

$Mn - \left(\frac{8M}{3}+\frac{M}{6}+\frac{M}{2}\right)$

OR $Mn - \left(\frac{16M}{6}+\frac{M}{6}+\frac{3M}{6}\right)$

$Mn - \frac{20M}{6}$

NEW MEAN $= \frac{Mn-\frac{20M}{6}}{n-3} = \frac{M(3n-10)}{3(n-3)}$